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GRAPHS OF THE FUNCTIONS Π AND Ψ

By E. P. R. Duval

The diagram on the opposite page consists of two graphs drawn accurately to scale. The curves printed in full lines represent (for values of x between $+3$ and -6) Gauss's Π-function, defined as an infinite product by the formula

$$\Pi(x) = \frac{1}{x+1} \prod_{n=2}^{n=\infty} \frac{n^{x+1}}{(n-1)^x (x+n)} = \Gamma(x+1).$$

The dotted curves represent (within the same limits) the function

$$\Psi(x) = \frac{d}{dx}\Big(\log \Pi(x)\Big).$$

The graphs were plotted by means of Gauss's tables for $\Pi(x)$ and $\Psi(x)$ for values of x between 0 and 1, and the relations

$$\Pi(x+1) = (x+1)\ \Pi(x),$$

$$\Psi(x+1) = \frac{1}{x+1} + \Psi(x).$$

The unit of length is the distance between two successive vertical asymptotes.

$\Pi(x)$ evidently has maximum or minimum points for all values of x for which $\Psi(x) = 0$. It was partly to illustrate this fact that the two graphs were drawn on the same pair of axes.

The only similar figure which the writer has seen is a somewhat rough graph of $\Gamma(x)$ in Maurice Godefroy's "La Fonction Gamma," page 63, or his "Théorie Elementaire des Séries," page 250.

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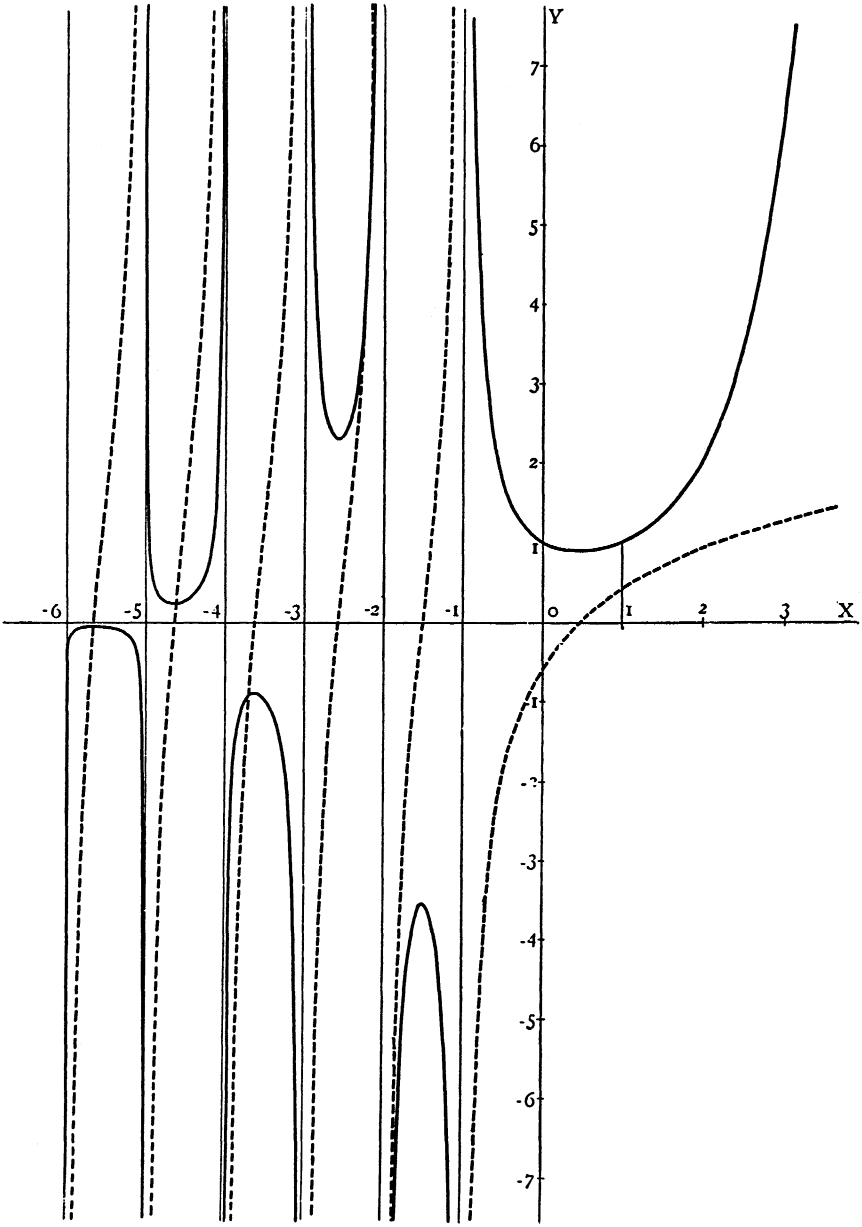
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